2022-LinearAlgebraII-minitest-confirmation.ipyn

問 1

In [12]:
```
using LinearAlgebra, SymPy
```

In [8]:
```
A = [ 1.0 3 5
2 4 6
8 10 12]
b = [ 1.0, 2, 8 ]
```

Out [8]:
```
3-element Vector{Float64}:
 1.0
 2.0
 8.0
```

行列 A は正則ではないことを確認.

In [10]:
```
det(A)
```

Out [10]:
```
0.0
```

手で求めた解

In [14]:
```
@vars p
x = [1, 0, 0] + p * [1, -2, 1]
```

Out [14]:
```
3-element Vector{Sym}:
 p + 1
  -2*p
     p
```

Ax を計算して，確かに b に一致するか？

In [15]:
```
A * x
```

Out [15]:
```
3-element Vector{Sym}:
 1.00000000000000
 2.00000000000000
 8.00000000000000
```

問 6

In [16]:
```
A = [ 13 -1 1
3 9 3
-1 1 11]
```

Out [16]:
```
3×3 Matrix{Int64}:
 13  -1   1
  3   9   3
```

```
 -1   1  11
```

In [18]:
```
eigen(A)
```

Out [18]:
```
Eigen{Float64, Float64, Matrix{Float64}, Vector{Float64}}
values:
3-element Vector{Float64}:
  9.000000000000005
 11.999999999999998
 12.0
vectors:
3×3 Matrix{Float64}:
 -0.301511   0.784465  0.22126
 -0.904534   0.588348  0.791279
  0.301511  -0.196116  0.570019
```

固有値 12 に対する固有ベクトルは平面中の2ベクトルなので一意性は無い．そこで確認．

In [19]:
```
v2, v3 = [ 1, 1, 0 ], [-1, 0, 1]
```

Out [19]:
```
([1, 1, 0], [-1, 0, 1])
```

In [21]:
```
A * v2
```

Out [21]:
```
3-element Vector{Int64}:
 12
 12
  0
```

In [22]:
```
A * v3
```

Out [22]:
```
3-element Vector{Int64}:
 -12
   0
  12
```

問 8

3^{100} とか桁数がひどいことになって通常計算だと値が正しくないので，多倍長計算を用いる．

In [30]:
```
A = [ big(5) -1
2 2 ]
```

Out [30]:
```
2×2 Matrix{BigInt}:
 5  -1
 2   2
```

Julia による計算

In [33]:
```
R8 = A^(100)
```

Out [33]:
```
2×2 Matrix{BigInt}:
 321387608851746517356319217335128874391464036629288356308075 1  …
-160693804425847489802123008101012614139243737251009072777937 5
 321387608851694979604246016202025228278487474502018145555875 0
-160693804425795952050049806967908968026267175123738862025737 4
```

In [29]:
```
a3, a4 = big(3)^100, big(4)^100
```

Out [29]:
```
(515377520732011331036461129765621272702107522001,
160693804425899027554196209234116260252220299378279283530137 6)
```

手で求めた解

In [34]:
```
B = a4 * [ 2 -1
           2 -1 ] +
a3 * [ -1 1
       -2 2 ]
```

Out [34]:
```
2×2 Matrix{BigInt}:
 32138760885174651735631921733512887439146403662928835630807 51  …
-16069380442584748980212300810101261413924373725100907277793 75
 32138760885169497960424601620202522827848747450201814555587 50
-16069380442579595205004980696790896802626717512373886202573 74
```

引き算してみて差を見てみる．完璧．

In [35]:
```
R8 - B
```

Out [35]:
```
2×2 Matrix{BigInt}:
 0  0
 0  0
```

問 9

In [36]:
```
A = [ 13 -1 1
3 9 3
-1 1 11]
```

Out [36]:
```
3×3 Matrix{Int64}:
 13  -1   1
  3   9   3
 -1   1  11
```

In [37]:
```
P = [ -1 1 -1
      -3 1  0
       1 0  1]
```

Out [37]:
```
3×3 Matrix{Int64}:
 -1  1  -1
 -3  1   0
  1  0   1
```

$P^{-1}AP$ を計算して，対角成分が 9, 12, 12 の対角行列になれば OK.

In [38]:
```
inv(P) * A * P
```

Out [38]:
```
3×3 Matrix{Float64}:
 9.0          -8.88178e-16   0.0
 0.0          12.0          -1.77636e-15
 1.77636e-15   0.0          12.0
```

問 10

Julia で計算した $\exp(A)$

In [39]:
```
As = exp(A)
```

Out [39]:
```
3×3 Matrix{Float64}:
      2.14305e5  -51550.6   51550.6
      1.54652e5    8103.08      1.54652e5
 -51550.6         51550.6       1.11204e5
```

手で計算した U'

In [40]:
```
a,b = exp(9), exp(12)
c = (a-b)

V = [ b-(c/3) -c c/3
c/3 a -c/3
-c/3 -c b+(c/3)]
```

Out [40]:
```
3×3 Matrix{Float64}:
      2.14305e5     1.54652e5  -51550.6
 -51550.6        8103.08        51550.6
  51550.6           1.54652e5       1.11204e5
```

これが手で計算した $\exp(A)$

In [41]:
```
U = V'
```

Out [41]:
```
3×3 adjoint(::Matrix{Float64}) with eltype Float64:
      2.14305e5  -51550.6   51550.6
      1.54652e5    8103.08      1.54652e5
 -51550.6         51550.6       1.11204e5
```

引き算して比較する．OK だろう．

In [42]:
```
As - U
```

Out [42]:
```
3×3 Matrix{Float64}:
  5.82077e-10  -3.63798e-11   1.30967e-10
  5.82077e-11   1.81899e-12  -5.82077e-11
 -1.23691e-10  -2.18279e-11  -1.16415e-10
```